principal value of the dielectric susceptibility of the axially symmetric particle along the axis of symmetry; **E**, intensity vector of the electric field; ω_{ik}, velocity vortex tensor; $\dot{\mathbf{n}}$, $\dot{n}_i$, and $\ddot{n}_i$, $\dot{\varphi}$, $\dot{\Theta}$, derivatives with respect to time; L, moment of inertia of dumbbell; I, moment of inertia of dumbbell with respect to the axis passing through the midpoint of the particle perpendicular to it; $\langle\ \rangle$, symbol indicating averaging by means of distribution function; $E_p=2m|2d_{km}d_{mk}|^{\frac{n-1}{2}}d_{ij}d_{ij}$, rate of dissipation of energy per unit volume of power-law dispersion medium in the absence of suspended particles; N_0, number of suspended particles per unit volume of suspension; $N_0\langle t_{ij}\rangle$, stress produced by the presence of N_0 suspended particles per unit volume of suspension; K, shear rate in simple shear motion; E, value of intensity vector of electric field; W, coefficient of rotational friction of the dumbbell; C, C_1, constants of integration; t, time; $\mu_a = (1/K)T_{(xy)}$, effective viscosity of suspension in simple shear flow; $\mu_p = m|K|^{n-1}$, viscosity of power-law dispersion medium in simple shear flow.

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RELATION BETWEEN HOMOGENEOUS AND INHOMOGENEOUS STRETCHING OF AN ELASTIC FLUID

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Homogeneous and inhomogeneous (steady) noninertial stretching of an elastic fluid is experimentally investigated, and a method is given for the calculation of one problem from experimental data on the other.

1. Homogeneous Stretching with Constant Force

The noninertial homogeneous stretching of cylindrical samples was first studied experimentally in [1, 2]. The experimental arrangement for stretching of this type is shown in Fig. 1a. One end of the test sample is fixed, and the other moves under the action of a constant force F. In the cylindrical coordinate system x, r, φ, the velocity components are

$$v_x = \varkappa(t)\,x;\ v_r = \frac{\varkappa(t)}{2}\,r;\ v_\varphi = 0. \tag{1.1}$$

Here x is the longitudinal coordinate measured from the point of fixing of the sample (Fig. 1a).

On the basis of the incompressibility condition for the fluid, the sample radius is

$$r(t) = r_0\varepsilon^{-1/2};\ \varepsilon = l/l_0. \tag{1.2}$$

Here r_0 and l_0 are the initial (t = 0) sample radius and length. Under the condition that the radial component of the stress tensor vanishes, and taking into account Eq. (1.2), the stress in the sample cross section is

$$\sigma = \sigma_{xx} = F/\pi r^2 = \sigma_0\varepsilon;\ \ \sigma_0 = F/\pi r_0^2. \tag{1.3}$$

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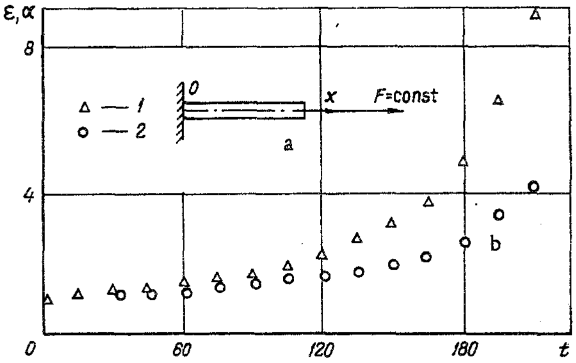

Fig. 1. a) Homogeneous stretching with a constant force F = const; b) dependence of the total ε (1) and elastic α (2) strain on the stretching time t, sec, for an initial stress $\sigma_0 = 6.3 \cdot 10^3$ N/m².

In the experiments described below, F, l_0, and r_0 are given; the dependence on t of the total l and residual l_r length is measured (l_r is the length to which the sample tends after the removal of the stress at time t). When σ = 0 the sample continues to contract as a result of the elastic energy accumulated in the course of preliminary stretching. The elastic longitudinal strain α is defined as follows

$$\alpha = l/l_r. \tag{1.4}$$

For a more detailed account of the kinetics of homogeneous extension, see [3].

2. Inhomogeneous Steady Stretching

The experimental arrangement for stretching of this type is shown in Fig. 2a. A cylindrical sample wound around a roller of radius R (and held by intrinsic adhesion) separates from the roller as the latter rotates at constant angular velocity ω_1 and is stretched by a constant force F.

Inhomogeneous stretching is technically simpler to obtain in the stretching of a fluid under the action of a force F produced by a capillary. But in this case shear strain in the capillary may distort the picture obtained on stretching, and this distortion has never been taken into account.

Introducing the cylindrical coordinate system z, r, φ, the z axis is directed along the sample being stretched, and the origin is chosen at the point at which the sample leaves the roller (Fig. 2a). Let r(z) and v_z be the radius and longitudinal velocity of the fluid sample. It is assumed that $|r'| = |dr/dz| < 1$ and that the total time derivative

$$\frac{d}{dt} \approx v_z \frac{\partial}{\partial z} \tag{2.1}$$

for the components of the stress σ and strain-rate **e** tensors. In this case [4, 5], the tensors σ and **e** are diagonal in terms of order up to r' (the nonzero component of the stress tensor is $\sigma = \sigma_{zz} = F/\pi r^2$) and, taking into account that $v_z|_{z=0} = V_1$, the velocity components are as follows:

$$v_z = V_1 + \int_0^z \varkappa(z)\,dz;\ v_r = -\frac{\varkappa(z)}{2} r;\ v_\varphi = 0.$$

Here v_z, v_r, and v_φ are the components of the velocity vector; ϰ is the strain rate.

Assuming that a polymer sample on a rotating roller is in an undeformed state up to the point z = 0 (whether this assumption is correct will be discussed later in considering the experimental results), and taking into account what has been said about inhomogeneous stratching with F = const, it is found that, as each cross section of the sample perpendicular to the z axis moves along the z axis, its deformation in the course of time t is the same as in the case of homogeneous stretching with the same force. The time t = 0 corresponds to z = 0, and $r|_{t=0} = r|_{z=0} = r_0$.

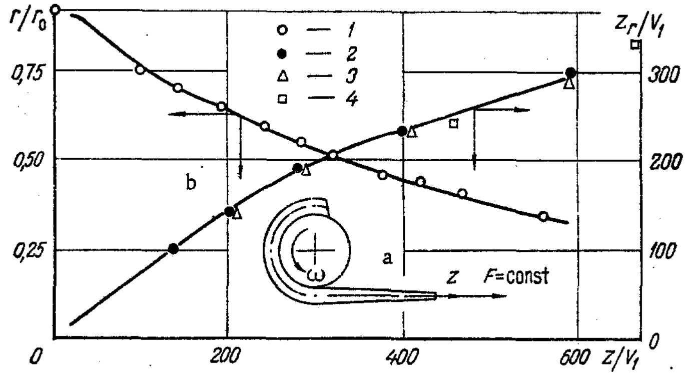

Fig. 2. a) Inhomogeneous stretching with F = const ($\sigma_0 = 6.3 \cdot 10^3$ N/m²); b) experimental dependence of r/r_0 (1) and z_r/V_1, sec, on z/V_1, sec: 2) $V_1 = 2.13 \cdot 10^{-4}$; 3) $1.07 \cdot 10^{-4}$; 4) $0.54 \cdot 10^{-4}$ m/sec. The continuous curves show calculated values.

The relation between the time of deformation t in homogeneous stretching and the coordinate z in inhomogeneous stretching is as follows:

$$z = \int_0^{t(z)} v_z(t)\,dt = V_1 \int_0^t \varepsilon(t_1)\,dt_1. \tag{2.2}$$

This relation is obtained from Eqs. (2.1) and (1.2) on taking into account that

$$v_z = q/\pi r^2 = V_1 (r/r_0)^2 = V_1 l/l_0 = V_1 \varepsilon. \tag{2.3}$$

Here $V_1 \approx \omega_1(R + r_0)$ is the flow velocity at z = 0.

The profile of the fluid sample in inhomogeneous stretching — the dependence r(z) — is calculated from Eqs. (1.2) and (2.2) using the known experimental dependence ε(t) obtained in homogeneous stretching.

The residual length z_r under inhomogeneous stretching is taken to mean the length which is obtained after the elastic recovery of the stretched sample, cut simultaneously at the points 0 and z (in the elastic recovery, $\sigma \cong 0$). The relation between z_r and the corresponding quantities in homogeneous stretching is as follows:

$$z_r = \int_0^z \frac{d\xi}{\alpha(\xi)} = V_1 \int_0^t \frac{\varepsilon(t)}{\alpha(t)}\,dt. \tag{2.4}$$

Here α(z) is the elastic strain. The derivation of Eq. (2.4) uses Eqs. (1.4), (2.1), and (2.3). The dependence $z_r(z)$ is calculated from experimental data on homogeneous stretching using Eqs. (2.4) and (2.2). Note that $z/z_r \neq \alpha(z)$ and does not depend on V_1, as is evident from Eqs. (2.2) and (2.4).

3. Experimental Investigation

Experiments to verify the above relations were carried out using a melt of P-20 polyisobutylene with initial viscosity $\eta \approx 1.1 \cdot 10^6$ N·sec/m² and equilibrium elastic modulus $G_e \approx 1.6 \cdot 10^3$ N/m² at 22°C.

Homogeneous stretching in the arrangement of Fig. 1a was carried out with $\sigma_0 = 6.3 \cdot 10^3$ N/m², produced by a load of ≈0.2 N. The initial length l_0 = 2-5 cm and the initial radius $r_0 \approx 0.3$ cm (both in homogeneous and inhomogeneous stretching). Cylindrical samples were prepared by rolling between two planes, with subsequent immersion in water for 1 h to remove any elastic strains. The sample density $\rho = 9.2 \cdot 10^2$ kg/m³. This method produces samples that differ from cylindrical shape by less than 3% over the radius. The length $l(t)$ was measured visually from a scale set alongside the sample. Note that at the moment at which loading begins ($t \gtrsim 0$) there is an abrupt but not very large increase in sample length. The change in l at small t was not measured. To measure l_r the sample was cut at different deformation times t and was immersed in water at 22°C for approximately 1 h after removal of the load but before measuring l_r [3].

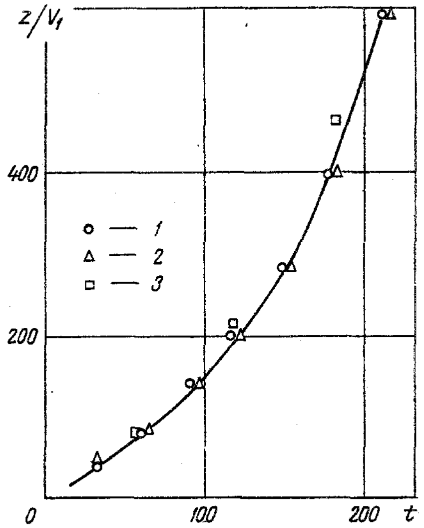

Fig. 3. Calculated (continuous curve) and experimental (points) dependence of z/V_1, sec, on the time t, sec: 1) $V_1 = 2.13 \cdot 10^{-4}$; 2) $1.07 \cdot 10^{-4}$; 3) $0.54 \cdot 10^{-4}$ m/sec.

Dependences of $\varepsilon = l/l_0$ (1) and $\alpha = l/l_r$ (2) on the time t, independent of l_0, are shown in Fig. 1b. Note that the dependence $\varepsilon(t)$ is naturally higher for an elastic fluid than for a Newtonian fluid of the same viscosity. Throughout the present work, the points shown in the figures correspond to results averaged over two or three experiments.

In the case of inhomogeneous stretching in the arrangement of Fig. 2a, the polymer was cut by a razor blade at $z = 0$ to separate it from the roller and stretched by a load of ≈ 0.2 N, as for homogeneous stretching. The roller radius $R = 4$ cm; the initial sample radius $r_0 \approx 0.3$ cm. In the experiments the sample velocity at $z = 0$ was $V_1 = 2.13 \cdot 10^{-4}$, $1.07 \cdot 10^{-4}$, and $0.54 \cdot 10^{-4}$ m/sec. To obtain the profile of the sample, it was photographed in a scale of 1:1. The elastic-recovery time for the measurement of z_r was chosen so as to agree with homogeneous stretching.

Steady stretching was investigated. The polymer on the roller began to deform a little (2-3 cm) before the point $z = 0$. The distance to which deformation extended along the sample around the roller increased with decrease in V_1.

Calculated curves of r/r_0 and z_r/V_1 on z/V_1, obtained from the experimental data of Fig. 1b and Eqs. (1.2), (2.2), and (2.4), are shown in Fig. 2b (curves 1 and 2). Here and below, no calculated dependences are given for small t, because of their inaccuracy.

The experimental points for r/r_0 (1) shown in Fig. 2b were obtained with $V_1 = 1.07 \cdot 10^{-4}$ m/sec. There is some discrepancy between the calculated and experimental data in the vicinity of $z = 0$; this is clearly because the deformation extends into the region $z < 0$ and also because $-dr/dz = r_0/2V_1 \cdot \varkappa/\varepsilon^{2/3}$ may be ≥ 1 when $z \approx 0$.

The experimental points for z_r/V_1 and z/V_1 (2, 3, 4) shown in Fig. 2b were obtained for various V_1. They form a single curve independent of V_1 and are in good agreement with the calculated curve — see Eqs. (2.2) and (2.4).

As an additional check, the dependence z(t) was also verified directly — see Eq. (2.2). In carrying out the experiments on the samples a groove was made perpendicular to the z axis, and the motion of this groove along the z axis ($t = 0 \leftrightarrow z = 0$) was observed. Experimental results for z/V_1 as a function of t for various V_1 are shown in Fig. 3. The maximum values measurable at these velocities were $z \approx 12.7$; 6.35; 3.2 cm. The sample radius at these lengths decreased by more than a factor of 3. The continuous curve in Fig. 3 shows the dependence obtained from Eq. (2.2) and Fig. 2. The dependence of z/V_1 on t is independent of V_1. For $V_1 = 0.54 \cdot 10^{-4}$ m/sec, some deviation of the points from the curve is observed; this is evidently because of the deformation of the sample on the roller at $z < 0$.

Note that the dependence r(z) plotted using data on homogeneous stretching with F = const taken from [2] has practically a discontinuity in the radius, with very slight subsequent

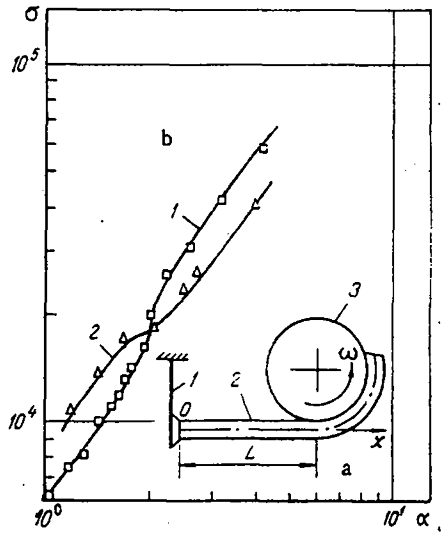

Fig. 4. a) Homogeneous stretching with $\varkappa$ = const; b) experimental dependence of tensile stress σ, N/m², on elastic strain α: 1) stretching with F = const at $\sigma_0 = 6.3 \cdot 10^3$ N/m²; 2) homogeneous stretching with $\varkappa = 1.2 \cdot 10^{-2}$ sec⁻¹.

change, at the point at which the crystallization process begins (this process is due to the tensile stress).

The strain rate in homogeneous stretching $\varkappa = (1/\varepsilon)(d\varepsilon/dt)$, after a sharp rise in the vicinity of t = 0, has a minimum with respect to t (t ≈ 90 sec), i.e., in inhomogeneous stretching $dv_z/dz = \varkappa(z)$ has the same minimum with respect to z.

Consider now the case when inhomogeneous steady stretching is produced not by a load but by a second roller, of radius R, rotating with angular velocity ω_2 ($\omega_2 > \omega_1$). The distance between the roller centers is s. The second roller gives the sample a constant velocity $v_z(s) = V_2 \approx \omega_2 R$. From the set of dependences of v_z/V_1 on z/V_1 for different σ_0 — these dependences are obtained from experimental data on ε(t) at different σ_0 and from Eqs. (2.2) and (2.3) — the dependence for which $v_z(s/V_1)/V_1 = V_2/V_1$ is found. Hence the required curve of ε(t) is determined — see Eq. (2.3) — and thereby also the stress and the profile of the steady inhomogeneous flow.

It is simple to consider the inverse problem, i.e., to derive data for homogeneous stretching from known results for inhomogeneous stretching. The inverse problem may be of interest in rheological investigations of fluids with relatively small viscosity, for which homogeneous stretching has not been possible.

The conversion from homogeneous stretching (which in many cases is simple to produce experimentally) to the inhomogeneous case (which is the more common in many engineering processes) may be used in the analysis of numerous phenomena — in the experimental investigation of the strength properties of fluids solidifying in the course of deformation [2, 5], in studying the effect of shear strain in a capillary on the subsequent stretching, in studying nonisothermal stretching, etc.

4. The Relation between Homogeneous Stretching at Constant Strain Rate $\varkappa$ = const and Inhomogeneous Stretching with F = const

Homogeneous stretching with $\varkappa$ = const (as distinct from the homogeneous stretching with F = const considered above) was carried out using the arrangement in Fig. 4a. The polyisobutylene sample 2 was fixed at one end to a sensor measuring the stretching force F and was wound at the other onto a roller 3 of radius R (R ≈ 4 cm) rotating at constant angular velocity ω. The distance between the roller center and the fixed end of the sample was L = 16 cm

(Fig. 4a). The velocity components were obtained from Eq. (1.1) for $\varkappa = \omega R/L$. The stress is $\sigma = F/\pi r^2$; the sample radius — see Eq. (1.1) — is

$$r = r_0 \exp(-\varkappa t/2). \tag{4.1}$$

The experimental temperature was 22°C. To check the constancy of $\varkappa$, the sample was photographed. The discrepancy between the experimental values of the radius and the results given by Eq. (4.1) was not more than 5%.

From the experimental data, the time dependences of the stress and elastic strain α — see Eq. (1.4) — were determined for fixed strain rate $\varkappa$. These dependences are monotonically increasing (for $\varkappa > 4.8 \cdot 10^{-4}$ sec^{-1} steady flow is not obtained).

For convenience in comparing homogeneous stretching with $\varkappa$ = const and inhomogeneous stretching with F = const, curves of $\sigma(\alpha)$ were plotted (Fig. 4b). As shown above the dependence $\sigma(\alpha)$ for inhomogeneous stretching with F = const coincides with the corresponding dependence for homogeneous stretching with F = const and is constructed from the data of Fig. 1 and from Eq. (1.3). Experimental curve 1 corresponds to homogeneous (or inhomogeneous) stretching under the action of a constant force with $\sigma_0 = 6.3 \cdot 10^3$ N/m^2. Experimental curve 2 corresponds to homogeneous stretching with $\varkappa = 1.2 \cdot 10^{-2}$ sec^{-1}. These curves do not coincide. At the point of intersection of the curves $\varkappa = 1.2 \cdot 10^{-2}$ sec^{-1} for the two types of deformation, i.e., at equal stress for these forms of deformation, α and $\varkappa$ must agree. This was also verified for other combinations of σ_0 and $\varkappa$. Note that the dependence $\sigma(\alpha)$ is not a single-valued function of α even for a single type of stretching.

For low-viscosity fluids the value $\eta^* = \sigma/\varkappa$ determined in inhomogeneous steady stretching is often (see [6], for example) taken to be the viscosity in steady homogeneous stretching with $\varkappa$ = const. As shown above, this may be incorrect.

Note finally that all the relations given above (in Secs. 1-3) may be used to investigate the formation of fibers and films from media with different rheological properties.

NOTATION

x (z), r, φ, cylindrical coordinates; v_x (v_z), v_r, v_φ, velocity components; t, time; $\varkappa$, strain rate; r_0 and l_0, initial radius and length of sample being stretched; l, length at time t; ε, total longitudinal strain; σ_{ij}, components of stress tensor σ; $\sigma = \sigma_{zz}$, stress in sample cross section; σ_0, initial stress; F, longitudinal stretching force; l_r and z_r, residual length of sample in homogeneous and inhomogeneous stretching; α, longitudinal elastic strain; q, flow rate; ρ, density; R, roller radius; ω_1, ω_2, ω, angular velocities of roller rotation; s, distance between rollers; L, base distance in stretching with $\varkappa$ = const; V_1, velocity of flow under inhomogeneous stretching at z = 0.

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